COLLATZ CONJECTURE

LESSON PLAN

GRADE:	SUBJECT:	DATE:

LESSON FOCUS AND GOALS:
To investigate what the Collatz conjecture is, practice substitution and then try some examples and learn about proof and why in maths it is so difficult

MATERIALS NEEDED:
Lots of paper
Projector (if possible)
These slides

LEARNING OBJECTIVES:
Improve algebraic understanding, improve understanding of proof

STRUCTURE / ACTIVITY:

The Set Up (5-10 minutes):
Begin with the instruction "choose a number between ten and twenty" Then ask, if it is even, half it. If it is odd, multiply by 3 and 1. Repeat this process until you notice something. Keep it to yourself. (if they finish quickly, ask them to try another number between 10 and 20)

For example - start with 13 - It is odd, so (x 3) + 1 = 40, then half it, half it again, and again because it is even, and we get 5. then, (x3) + 1, and we get 16, half, half, half and half, and we get to 1.

When students get to 1 (and they should, so if not check their understanding) the process repeats and they will always return to 1.

The Investigation (20 - 30 minutes)

Make a sweeping statement - all numbers between 10 and 20 end up on 1.
Depending on what numbers were chosen - you hopefully will be able to PROVE that this is true. This is called **proof by exhaustion** - checking that **every** possible number works.

Then ask the most important question. What next?
Depending on your class ability or their exposure to this idea, you may need to coax them to ask either of these two question:
Does it work for all numbers? (or something similar)
What numbers get to 1 quickest?

Either of these can spin off into investigations. At this point you don't need to go into details about conjectures and what they are.

The first one is the conjecture, that all numbers will end on one. Choose a number between 20 and 100. Work on it with your class (some take a long time to get to 1 but they all do)

COLLATZ CONJECTURE

LESSON PLAN PAGE 2

THE INVESTIGATION (CONTINUED)

After five - ten minutes of discussion. Then discuss the Collatz Conjecture:

The conjecture is that no matter what value of starting number n, the sequence will always reach 1. It was named after mathematician Lothar Collatz in 1937. and is called a conjecture, rather than a theorem, because it is unproven. This is because it cannot be proven by exhaustion (too many numbers to check every single one), yet no-one has ever found a proof by contradiction (in other words no one has ever found a number that doesn't go to 1).

This is now one of the top 10 unsolved problems in mathematics. It has cash prizes if someone can find a number that doesn't go to 1. (Cue excited grins!). Paul Erdös, a prolific mathematical researcher, once said "Mathematics may not be ready for such problems".

Then let them loose. How many numbers can they check before the lesson is up? Here is a tip - create a drawing - like a mind map - of how numbers connect. Drawing a picture is an important strategy in solving problems. For example, someone works out that 24 drops down to 1 in 10 steps. So the number 48 links to 24, as will 96. so you have a chain of numbers that all follow the same steps. if you ever land on 7, you know that from that point it takes 16 steps to get to 1.

The Follow Up

Give time for the write up (below), about ten minutes (depending on age). There are some other investigations and follow up questions. As mentioned earlier, a great challenge is to find the shortest chain starting with a number between 20 to 100) (answer 32, powers of 2 are the quickest chains) and the longest chain.
You can also try negative numbers which remarkably has three numbers it could get stuck on, not just -1.
Finally - once they have the idea and have done plenty of maths - let them use technology to help! The Scratch project linked here can be used to workout how many steps numbers take to get to 1.

But how do you know? What makes you SURE you have the largest / smallest chain?

ASSESSMENT:

Have the students create a simple paragraph explaining the Collatz Conjecture, and two examples.
Finally ask them to write down if they think this will ever be proven or disproven and why.
The communication is key here - can they explain efficiently how the process works?

Mini Task: In less than 12 words explain what the word conjecture means!